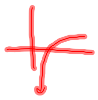
24. $\lim_{x\to 0^+} (\sin x)^x = y \qquad 0^0$

$\lim_{x\to 0^+} \ln (\sin x)^x = \ln y$

$\lim_{x\to 0^+} \overset{0}{x} \overset{(-\infty)}{\ln(\sin x)} = \ln y$



$\lim_{x\to 0^+} \dfrac{\ln(\sin x)}{\frac{1}{x}} \quad \dfrac{-\infty}{\infty}$

$\lim_{x\to 0^+} \dfrac{\frac{1}{\sin x}\cdot \cos x}{-\frac{1}{x^2}} = \lim_{x\to 0^+} \dfrac{-\cos x \cdot x^2}{\sin x} \quad \dfrac{0}{0}$

$-\lim_{x\to 0^+} \dfrac{\overset{\to 0}{\cos x \cdot 2x + x^2(-\sin x)}}{\underset{\to 1}{\cos x}} = 0$

$0 = \ln y$

$y = e^0 = 1$

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45. $\lim_{x\to 0^+} (1+x)^{\frac{1}{x}} = y$

$\lim_{x\to 0^+} \ln(1+x)^{\frac{1}{x}} = \ln y$

$\dfrac{0}{0} \quad \lim_{x\to 0^+} \dfrac{\ln(1+x)}{x} = \ln y$

$\lim_{x\to 0^+} \dfrac{\frac{1}{1+x}\cdot 1}{1} = \ln y$

$1 = \ln y$

$e^1 = y$

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55.

$$f(x) = \begin{cases} \dfrac{9x - 3\sin 3x}{5x^3} & x \neq 0 \\ c & x = 0 \end{cases}$$

$\frac{0}{0}$ $$\lim_{x \to 0} \frac{9x - 3\sin 3x}{5x^3} =$$

$\frac{0}{0}$ $$\lim_{x \to 0} \frac{9 - 9\cos 3x}{15x^2} =$$

$\frac{0}{0}$ $$\lim_{x \to 0} \frac{27\sin 3x}{30x}$$

$$\lim_{x \to 0} \frac{81\cos 3x}{30} = \frac{81}{30}$$

II $f(0) = \frac{81}{30} = c$

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8.3 Relative Growth Rates

compare 2 functions to determine which grows faster (slower)

if $\lim_{x \to \infty} \frac{f(x)}{g(x)} = \infty$ then f grows faster

$\lim_{x \to \infty} \frac{f(x)}{g(x)} = 0$ then g grows faster

$\lim_{x \to \infty} \frac{f(x)}{g(x)} = L$ $0 < L < \infty$ same rate

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$f(x) = 2x \quad g(x) = x^2$

$$\lim_{x\to\infty} \frac{2x}{x^2} = 0$$

$$\lim_{x\to\infty} \frac{x^2}{2x} = \infty$$

$x^2$ grows faster

---

$e^x, \; x^3$

$$\lim_{x\to\infty} \frac{e^x}{x^3} = \lim_{x\to\infty} \frac{e^x}{3x^2} = \lim_{x\to\infty} \frac{e^x}{6x}$$

$$= \lim_{x\to\infty} \frac{e^x}{6} = \infty$$

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$\ln x, \quad 7x \qquad \lim_{x\to\infty} \frac{\ln x}{7x}$

$$\lim_{x\to\infty} \frac{\frac{1}{x}}{7} = 0$$

so $\ln x$ slower than $7x$

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$y = x$

$y = x + \sin x$

$y = e^x$

$y = e^{2x}$ $\lim_{x\to\infty} \frac{e^x}{e^{2x}} =$

$\lim_{x\to\infty} \frac{e^x}{(e^x)^2} = 0$

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Ex 5

$y = \sqrt{x^2+5}$ — like $y=x$

$y = (2\sqrt{x}-1)^2$ like $y=x$

compare both with $y=x$

$\lim_{x\to\infty} \frac{\sqrt{x^2+5}}{x}$ $\lim_{x\to\infty} \frac{(2\sqrt{x}-1)^2}{x}$

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